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✓ STL-TR-60-0000-19


A Measurement of the Electrical Conductivity
of Ionized Gases

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By

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December 31, 1960

Approved


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ABSTRACT

Two methods were used in attempts to measure the electrical conductivity of various substances, in particular that of a plasma jet. ~~In the first set of experiments,~~ Transients flowing in a system of coupled circuits were subtracted in such a way that the difference signal is due only to the presence of the conducting material. The amplitude and time decay of the signal are calculable functions of the conductivity. This method worked well on graphite, ~~($\sigma \sim 1000 \text{ mho/cm}$)~~, but was too insensitive for aqueous salt (NaCl) solutions. ~~($\sigma \sim 1 \text{ mho/cm}$)~~. Therefore, it cannot easily be applied to the case of an air arc jet exhaust. ~~expected value of $\sigma \sim 1 \text{ mho/cm}$?~~

The steady-state AC measurement technique was tried next. ~~In these experiments~~ Resonance properties of a coil coupled to the conducting material were measured and compared to the corresponding properties of the same coil in the absence of the conducting substance. This method was used on salt water in the frequency range of 1-10 mc. It was found to be sufficiently sensitive that the accuracy of the conductivity measurement was expected to be about 10%. Two problems were encountered, however, that were not solved in the time available for this project. First, the salt water was found to have an appreciable capacitive reactance, which made it unsuitable as a calibrating medium. Second, grounding of one end of a tube filled with a sample of salt water caused undesirable changes in readings. It was necessary to ground one end of the sample in order to simulate properly the plasma jet which is grounded at the nozzle.

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FOREWORD

This report summarizes research carried out under a project entitled, "The Investigation of the Physical Properties of Ionized Gases," MJO 9813, which is part of the Space Technology Laboratories' General Research Program. The major part of this project was to be the development of a technique for measuring the electrical conductivity of ionized gases in steady flow. There was particular interest in the measurement of the conductivity in the exhaust of an arc jet.

Nominally, the project was to continue until December 31, 1960, having started on January 1, 1960. As a result of transfer of personnel to the Aerospace Corporation, this work was discontinued at the end of August 1960. The report, therefore, covers only the work done up to that time, and the project must be considered incomplete.

ABSTRACT

Two methods were used in attempts to measure the electrical conductivity of various substances, in particular that of a plasma jet. In the first set of experiments, transients flowing in a system of coupled circuits were subtracted in such a way that the difference signal is due only to the presence of the conducting material. The amplitude and time decay of the signal are calculable functions of the conductivity. This method worked well on graphite ($\sigma \sim 1000$ mho/cm), but was too insensitive for aqueous salt (NaCl) solutions ($\sigma \sim 1$ mho/cm). Therefore, it cannot easily be applied to the case of an air arc jet exhaust (expected value of $\sigma \sim 1$ mho/cm).

The steady-state AC measurement technique was tried next. In these experiments resonance properties of a coil coupled to the conducting material were measured and compared to the corresponding properties of the same coil in the absence of the conducting substance. This method was used on salt water in the frequency range of 1-10 mc. It was found to be sufficiently sensitive that the accuracy of the conductivity measurement was expected to be about 10%. Two problems were encountered, however, that were not solved in the time available for this project. First, the salt water was found to have an appreciable capacitive reactance, which made it unsuitable as a calibrating medium. Second, grounding of one end of a tube filled with a sample of salt water caused undesirable changes in readings. It was necessary to ground one end of the sample in order to simulate properly the plasma jet which is grounded at the nozzle.

SYMBOLS

| | |
|--------------|--|
| a | radius of primary coil |
| A | coefficient in current transient |
| B | magnetic field |
| C | capacity |
| D | coefficient in current transient |
| E | electric field |
| F | coefficient in current transient |
| G | " " " " |
| i | current |
| J | $\sqrt{-1}$ |
| k | coupling constant |
| L | inductance |
| M | mutual inductance |
| P | reciprocal time constant |
| Q | quality factor of coil = $\omega L/R$ |
| r | radius |
| R | resistance |
| t | time |
| v | jet velocity |
| V | battery voltage |
| X | reactance |
| Z | impedance |
| γ | defined by: $p_1 = -\pi_1 \cdot \{1+\gamma\}$ propagation constant in wave equation |
| δ | $= 1 - (1 + \frac{1}{2}\epsilon)(1 + \epsilon)^{-1}$ |
| ϵ | $= \sqrt{1 + 4k^2\pi_1\pi_3(\pi_3 - \pi_1)^{-2}} - 1$ |
| ϵ_0 | permittivity |
| μ | permeability |
| π | single coil reciprocal time constant = R/L |
| σ | conductivity |
| ω | radian frequency |

Subscripts

| | |
|---|--|
| 1 | primary coil |
| 2 | search coil |
| 3 | plasma jet or sample |
| c | conductivity sample (salt water, plasma,...) |
| r | radial component |
| z | axial component |

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I. INTRODUCTION

There is currently a great interest in the application of magnetohydrodynamics to propulsion and power generation. In most cases the working fluid in MHD devices is an ionized gas. It appears that the electrical conductivity of such a gas is a parameter playing an important role in the performance of such devices. For many pure gases the conductivity can be calculated (in equilibrium) or measured using shock tube techniques,¹ but this is not possible for the arc jet or the alkali-seeded gases of interest in magnetohydrodynamic applications. It is difficult to calculate the effects of impurities and non-equilibrium on the conductivity of the arc jet, and, similarly, it is difficult to take into account nonequilibrium effects in seeded gases. It is, therefore, desirable to have available a method for measuring the conductivity of a plasma jet in steady flow. This report describes one such method in which appropriate electrical measurements are made on a coil coupled to the plasma jet.

The application of an inductive coupling technique to the measurement of electrical conductivity is not new. The steady state method has been used by Blackman.² The results reported by Blackman, however, are questionable. The transient method has been applied successfully to the measurement of the conductivity of metals.³ This method cannot be applied directly to our purpose because the conductivity of ionized gases results in time constants that are too small to measure conveniently.

The object of this project, then, was to develop a reliable method of measuring the electrical conductivity of high temperature gases using the inductive coupling technique. The method would either have to be capable of yielding an absolute measurement or make use of calibrating substances whose electrical properties are similar to those of the gases of interest. Both the transient and steady state AC methods were considered and tried. The following pages describe these methods in some detail.

II. DESCRIPTION OF EXPERIMENTS

1. The Transient Method

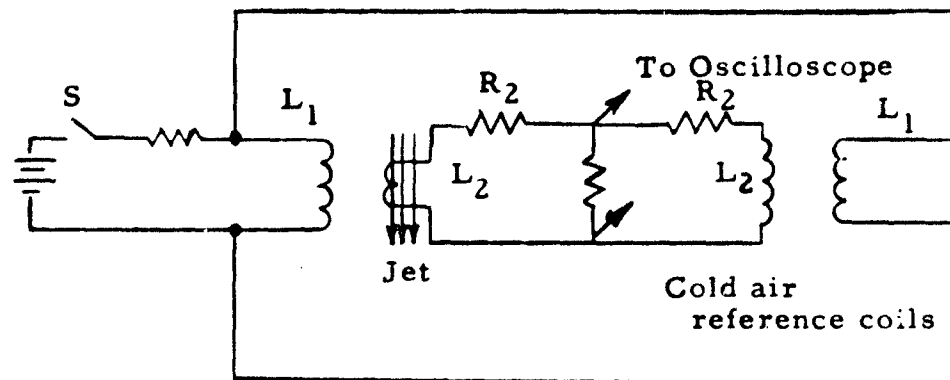


Figure 1. Circuit for Transient Measurement

Figure 1 represents the circuit used in first attempts to apply the transient method to a conductivity measurement. The coils are balanced as accurately as possible. They are connected in such a way that the currents produced by the two coils L_2 tend to cancel each other. The difference signal applied to the oscilloscope terminals represents the effect of the ionized jet. The oscilloscope trace is the voltage across the center resistor as a function of time after the closing of switch S.

(a) Analysis

The behavior of the circuit of Figure 1 can be treated by standard methods if one assumes an equivalent circuit for the plasma jet, consisting of an inductance and resistance in series, coupled to both L_1 and L_2 . It is also possible to analyze this circuit more rigorously by taking into account the current distribution through the plasma. However, for the present purpose the lumped circuit analysis will show the important characteristics of this method with sufficient accuracy.

It is assumed in the following that the circuit senses no effect due to the motion of the plasma jet. For this to be true, the EMF due to the radial magnetic field must be small compared to the field induced by the rate of change of axial flux. Quantitatively,

$$\text{Motional EMF} = E_m = vB_r$$

$$\text{Induced field} = E_{\text{ind}} = \frac{1}{2\pi r} \frac{d\phi}{dt} = \frac{1}{2\pi r} (\pi r^2) \frac{dB_z}{dt} = \frac{r}{2} p B_z$$

where

v = jet velocity

p = typical inverse time constant

Thus the condition that the effects of the motion be negligible is that

$$\frac{E_m}{E_{\text{ind}}} = \frac{2vB_r}{prB_z} \ll 1. \quad (1)$$

Formulas for B_r/B_z as a function of r and z are not conveniently available for multiturn coils.* But this information can be found in Smythe⁴ for a single turn coil. Qualitatively, one can reason that applying such a result to a multiturn coil is very likely conservative. Carrying out the calculations according to Smythe, one finds that the maximum value of $\frac{B_r}{r B_z}$ is about a^{-1} , where a is the coil radius. Using a jet velocity of $5,000 \text{ ft}^2/\text{sec} = 1.5 \times 10^5 \text{ cm/sec}$, a radius of 1 cm for the coil L_1 , and a typical value for

* At the time of writing it is learned that there is a machine program available, developed by the Physical Research Laboratory of STL, which computes the field of multiturn coils.

p of 10^6 sec^{-1} , Eq. (1) yields a ratio of 0.3. This shows that one has to be careful in the choice of parameters. But it should not be forgotten that the maximum value used above occurs only in a region defined by a narrow range of the variables r and z . Since one measures the integrated effect of the currents flowing in the entire plasma, the errors introduced by neglecting the effect of the motion must be negligible even for the values assumed above.

The results of the transient analysis of the circuit shown in Figure 2 are given below. This circuit is equivalent to that shown in Figure 1 except that L_2 is shown open-circuited.

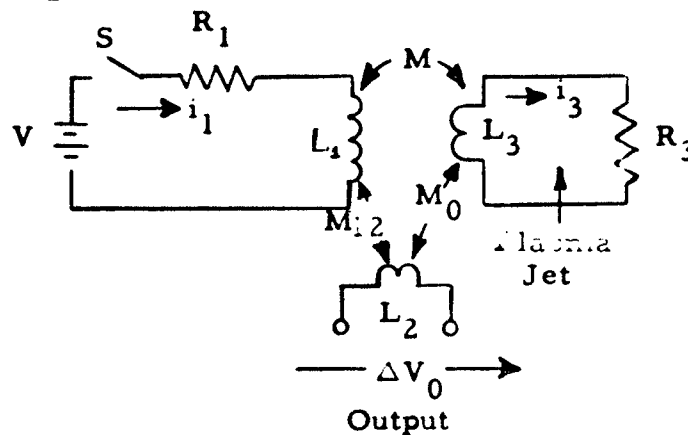


Figure 2. Equivalent Circuit for Transient Measurement

If switch S is closed at time $t = 0$, the currents flowing in the two loops are given by the following expressions:

$$i_1 = \frac{V}{R_1} \left\{ 1 - (1-\delta) e^{p_1 t} - \delta e^{p_3 t} \right\} \quad (2)$$

$$i_3 = \frac{MV}{L_1 L_3 (\pi_3 - \pi_1) (1+\epsilon)} (e^{p_1 t} - e^{p_3 t}) \quad (3)$$

The following definitions are used:

$$k^2 = \frac{M^2}{L_1 L_3}$$

$$p_1 = - \frac{1}{(1-k^2)} \left\{ \pi_1 - \frac{1}{2} \epsilon (\pi_3 - \pi_1) \right\}$$

$$p_3 = - \frac{1}{(1-k^2)} \left\{ \pi_3 + \frac{1}{2} \epsilon (\pi_3 - \pi_1) \right\}$$

$$\epsilon = \sqrt{1 + \frac{4R_1 R_3 M^2}{(R_1 L_3 - R_3 L_1)^2}} - 1$$

$$= \sqrt{1 + \frac{4k^2 \pi_1 \pi_3}{(\pi_3 - \pi_1)^2}} - 1$$

$$\epsilon \approx \frac{2k^2 \pi_1 \pi_3}{(\pi_3 - \pi_1)^2}$$

$$\delta = 1 - \frac{1+0.5\epsilon}{1+\epsilon}$$

$$\delta \approx \frac{1}{2} \epsilon$$

$$\delta \approx \frac{k^2 \pi_1 \pi_3}{(\pi_3 - \pi_1)^2}$$

$$\pi_1 = \frac{R_1}{L_1}$$

$$\pi_3 = \frac{R_3}{L_3}$$

$$p_1 = - \pi_1 \{1 + \gamma\}$$

$$\gamma \approx - \frac{k^2}{1-k^2} \frac{\pi_1}{\pi_3 - \pi_1}$$

The derivatives are

$$\frac{di_1}{dt} = -\frac{V}{R_1} \left\{ p_1 (1-\delta) e^{p_1 t} + p_3 \delta e^{p_3 t} \right\} \quad (4)$$

$$\frac{di_3}{dt} = \frac{MV}{L_1 L_3 (\pi_3 - \pi_1)(1+\epsilon)} \left\{ p_1 e^{p_1 t} - p_3 e^{p_3 t} \right\} \quad (5)$$

The total output difference signal is

$$\begin{aligned} \Delta V_o = M_{12} \frac{V}{R_1} & \left\{ p_1 (1-\delta) e^{p_1 t} + \pi_1 e^{-\pi_1 t} + p_3 \delta e^{p_3 t} \right\} \\ & + M_o \frac{MV}{L_1 L_3 (\pi_3 - \pi_1)(1+\epsilon)} \left\{ p_1 e^{p_1 t} - p_3 e^{p_3 t} \right\} \end{aligned} \quad (6)$$

The part of the voltage ΔV_o due to the primary currents is

$$\Delta V_{o1} = M_{12} \frac{V}{R_1} \left\{ p_1 \left[(1-\delta) e^{p_1 t} - (1-\gamma) e^{p_1 (1-\gamma)t} \right] + p_3 \delta e^{p_3 t} \right\},$$

which can be written, approximately

$$\begin{aligned} \Delta V_{o1} & \simeq M_{12} \frac{V}{R_1} \left\{ p_1 \left[(1-\delta) - (1-\gamma)(1-p_1 \gamma t) \right] e^{p_1 t} + p_3 \delta e^{p_3 t} \right\} \\ & \simeq M_{12} \frac{V}{R_1} \left\{ p_1 \left[\gamma(1+p_1 t) - \delta \right] e^{p_1 t} + p_3 \delta e^{p_3 t} \right\} \\ & \simeq M_{12} \frac{V}{R_1} \left\{ p_1 \gamma \left[1 + (1-k^2) \frac{\pi_3}{\pi_3 - \pi_1} + p_1 t \right] e^{p_1 t} + p_3 \delta e^{p_3 t} \right\} \\ & = A e^{p_1 t} + D e^{p_3 t} \end{aligned}$$

At $t = 0$ the amplitude of that part of the exponential, $\exp(p_1 t)$, due to the primary currents is

$$\begin{aligned} A & = M_{12} \frac{V}{R_1} p_1 \gamma \left[1 + (1-k^2) \frac{\pi_3}{\pi_3 - \pi_1} \right] \\ & = NM_{12} \frac{V}{R_1} p_1 \gamma, \end{aligned}$$

where N is a number between 1 and 2, since the second term in the brackets $\left[\right]$ can go from 0 to 1, depending on the ratio π_3/π_1 . Similarly,

$$\Delta v_{03} = Fe^{p_1 t} + Ge^{p_3 t}.$$

The amplitude of that part of the exponential, $\exp(p_1 t)$, due to the current i_3 (the plasma current) is

$$F = \frac{M_o M v p_1}{L_1 L_3 (\pi_3 - \pi_1) (1 + \epsilon)}.$$

Making appropriate substitutions this can be written

$$F = - \frac{k_o}{k_{12} k} \frac{(1 - k^2)}{(1 + \epsilon)} M_{12} \frac{V}{R_1} p_1 \gamma,$$

where the k 's are the coupling coefficients corresponding to the mutual inductances of Figure 2. Thus, the ratio of the two parts of the exponential $\exp(p_1 t)$ is approximately

$$\left| \frac{F}{A} \right| \approx \frac{k_o}{k_{12} k}.$$

Experimentally, k_{12} represents loose coupling, k_o fairly tight coupling, and k again represents loose coupling, as implied by the approximations made above. Thus, the ratio is of the order of 10; and the output ($\exp [p_1 t]$) is determined mostly by the direct coupling between plasma and output coils. A similar result can be obtained for the other exponential ($\exp [p_3 t]$).

Rewriting the significant part of the output signal once again, i.e., the second term of Eq. (6), we obtain

$$\Delta v_o \approx \Delta v_{03} = Fe^{p_1 t} + Ge^{p_3 t}$$

or,

$$v_o \approx \frac{k_o k}{(1 + \epsilon)} \sqrt{\frac{L_2}{L_1}} \frac{V}{(\pi_3 - \pi_1)} \left\{ p_1 e^{p_1 t} - p_3 e^{p_3 t} \right\}.$$

Considering experimental values of

$$k \sim 0.01 - 0.10$$

$$k_0 \sim 0.5$$

$$\frac{L_2}{L_1} \sim 0.01 - 0.10 ,$$

ΔV_0 can be written

$$\Delta V_0 \approx (0.0005 - 0.015)V \left\{ \frac{p_1}{\pi_3 - \pi_1} e^{p_1 t} - \frac{p_3}{\pi_3 - \pi_1} e^{p_3 t} \right\} , \quad (7)$$

where the first bracket represents the range of the numerical multiplier.

If p_3 is too large for the oscilloscope to register the exponential $\exp(p_3 t)$, and, if $p_3 \gg p_1$, then Eq. (7) shows that the output is of the order of millivolts or less if V is of the order of volts; and the output is proportional to the plasma conductivity. If $\exp(p_3 t)$ can be observed on the oscilloscope, then the maximum value of the bracket $\{ \}$ (Eq. 7) is unity; thus, the maximum observable ΔV_0 is about 0.015V.

(b) Experimental Results

When a graphite rod was used in place of the plasma jet in the circuit of Figure 1, a sizable signal was obtained. Thus, for a material whose conductivity is of the order of that of graphite ($\sim 1,000$ mho/cm) this method is easily applied to the measurement of conductivity. On the other hand, when a glass tube filled with salt water was used to simulate the plasma, the difference signal was too small compared to the noise level and compared to the signal due to the residual differences between the reference and measurement circuits. In these experiments, π_1 was of the order of 10^5 , π_3 (salt water) of the order of 10^8 . It was reasoned from Eq. (7) that if π_1 were increased the sensitivity should be materially improved. π_1 was increased by using a smaller value of L_1 . The output signal, however, was still too small and too noisy. The reason for this is probably twofold: first, the switching was no longer fast enough compared to the exponential decay $\exp(p_1 t)$; second, the oscilloscope reproduced a signal which showed both attenuation and distortion, because the increased p_1 now corresponds to frequencies (~ 10 mc) beyond the range of the instrument.

It is probable that all of these experimental difficulties could be overcome by sufficiently sophisticated techniques. It is clear, however, that the desired information can be obtained with much simpler techniques if the steady-state AC method is used. It was therefore decided to try this method.

2. The Steady-State AC Method

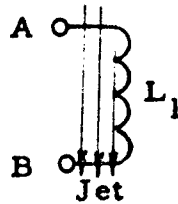


Figure 3. Steady-State AC Measurement Circuit

The steady-state AC measurement is very simple in principle. The circuit is shown in Figure 3. Appropriate measurements are made at the terminals AB, comparing the coil in air with the coil coupled to the plasma jet. Frequencies are used such that the expected "skin depth" of the plasma is large compared to the radius of the plasma jet. The measured quantity could be either RF impedance or the effective Q of the coil. In principle, the impedance bridge has the same sensitivity as a Q-meter. But in practice it was found that the impedance bridge was not sufficiently sensitive in the required range of resistance and reactance. Most of the experiments were therefore concerned with Q-meter measurements. It should be pointed out here that attempts were made to use impedance bridge results as a check on Q-meter measurements. In these attempts the circuit used in the bridge measurement was modified so as to increase the sensitivity of the bridge in the range of interest. Satisfactory results were not obtained, however, because of lack of time. For reference, the principal features of a Q-meter circuit are shown in Figure 4.

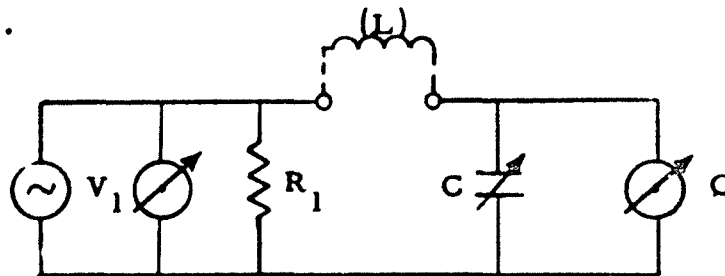


Figure 4. Q-Meter Circuit

The unknown coil is connected at L. The voltmeter Q reads Q directly if the generator output is adjusted such that V_1 reads 1 volt, and if C is adjusted for resonance. This can be seen by inspection. Q is defined here as $(\omega L/R)_{\text{coil}}$.

(a) Analysis

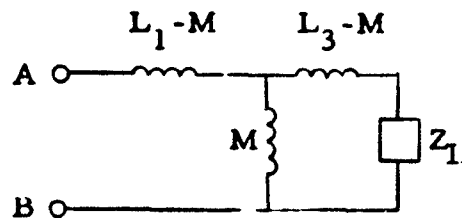


Figure 5. Equivalent Circuit for AC Measurement

Figure 5 shows the simplest equivalent circuit for the measurement. The notation is the same as that of Figure 2. It is an easy matter to derive the impedance "coupled" or "reflected" into the primary, i.e., the change in impedance Z_{AB} due to the plasma

$$\Delta Z_{AB} = \Delta Z = -\frac{Z_m^2}{Z_3}, \quad (8)$$

where

$$Z_m = j\omega M$$

$$Z_3 = Z_L + j\omega L_3 = \text{total secondary loop impedance.}$$

The quantity measured by the Q-meter is:

$$Z - Z_0 = \Delta Z = \Delta R + j\Delta X$$

$$\Delta R = R - R_0$$

$$\Delta X = X - X_0$$

where

R = resistance of coil coupled to plasma jet

X = reactance of coil coupled to plasma jet

R_0 = resistance of coil in air

X_0 = reactance of coil in air .

ΔX is computed from the capacitance readings, and ΔR can be found from measured quantities as follows:

$$\Delta R = \frac{\Delta Q \cdot X_0 + \Delta X \cdot Q_0}{Q_0 Q}$$

where $Q = Q_0 - Q$

and $Q_0 = Q$ of coil in air

$Q = Q$ of coil coupled to plasma jet

From Eq. (8)

$$\frac{\Delta R}{\Delta X} = - \frac{R_3}{X_3}$$

where $R_3 = \text{Re}(Z_3)$

$X_3 = \text{Im}(Z_3)$

In the simple case for which Z_L represents just the equivalent resistance of the plasma,

$$- \frac{\Delta R}{\Delta X} = \frac{R_3}{\omega L_3} = \frac{\pi_3}{\omega} .$$

To a first approximation π_3 is the first order inverse time constant of the plasma jet, and can be computed taking into account the distribution of the current flow. π_3 is a known function of the dimensions and the plasma conductivity, as follows:

$$\frac{1}{\pi_3} = 2.17 (r_c)^2 \sigma \times 10^{-9} \quad (9)$$

where r_c = radius of sample, cm

σ = conductivity, mho/cm

$\frac{1}{\pi_3}$ = time constant, sec.

Thus the Q-meter measurements could be used to calculate the plasma conductivity, through Eq. (9). It should also be possible to make a more precise

calculation of π_3 by a method analogous to that used by C. P. Bean et al.³ Ignoring the difficulties discussed below, it would be necessary to solve the wave or eddy-current equation in cylindrical coordinates as follows:

$$\nabla^2 \underline{B} = \Gamma^2 \underline{B}$$

where

$$\begin{aligned} \Gamma^2 &= j\omega\mu (\sigma + j\omega\epsilon_0) \\ &\approx j\omega\mu\sigma \end{aligned}$$

Using the fact that there is no variation with ϕ , the scalar wave equations for the components are found to be

$$\nabla^2 B_r = \Gamma^2 B_r + \frac{1}{r^2} B_r$$

and $\nabla^2 B_z = \Gamma^2 B_z$.

The solutions could be found by matching the well-known general solutions of the eddy-current equation at the plasma-air boundary and at the coil boundary

(b) Experimental Results

In order to verify the preceding theory, an aqueous salt (NaCl) solution was used to simulate the plasma jet. Salt water was used (as in the transient method) because its conductivity is known or can be measured by other, simpler techniques, and because the conductivity of a saturated solution is approximately that to be expected of the air jet used in this laboratory. Solutions of various strengths were prepared, and their conductivities measured using an RF impedance bridge. The salt water was poured into a channel which was cut to the shape of a rectangular loop as shown in Figure 6. In the frequency range of 1-10 mc, the measured resistive component agreed well with published conductivity tables (see Figure 7). An appreciable capacitive reactance component (series) was also measured. For a loop length of approximately 3 ft the measured capacitance of the saturated salt solution was in the range 0.01-0.1 μ f.

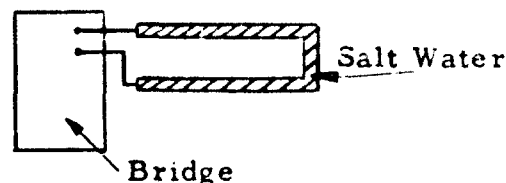
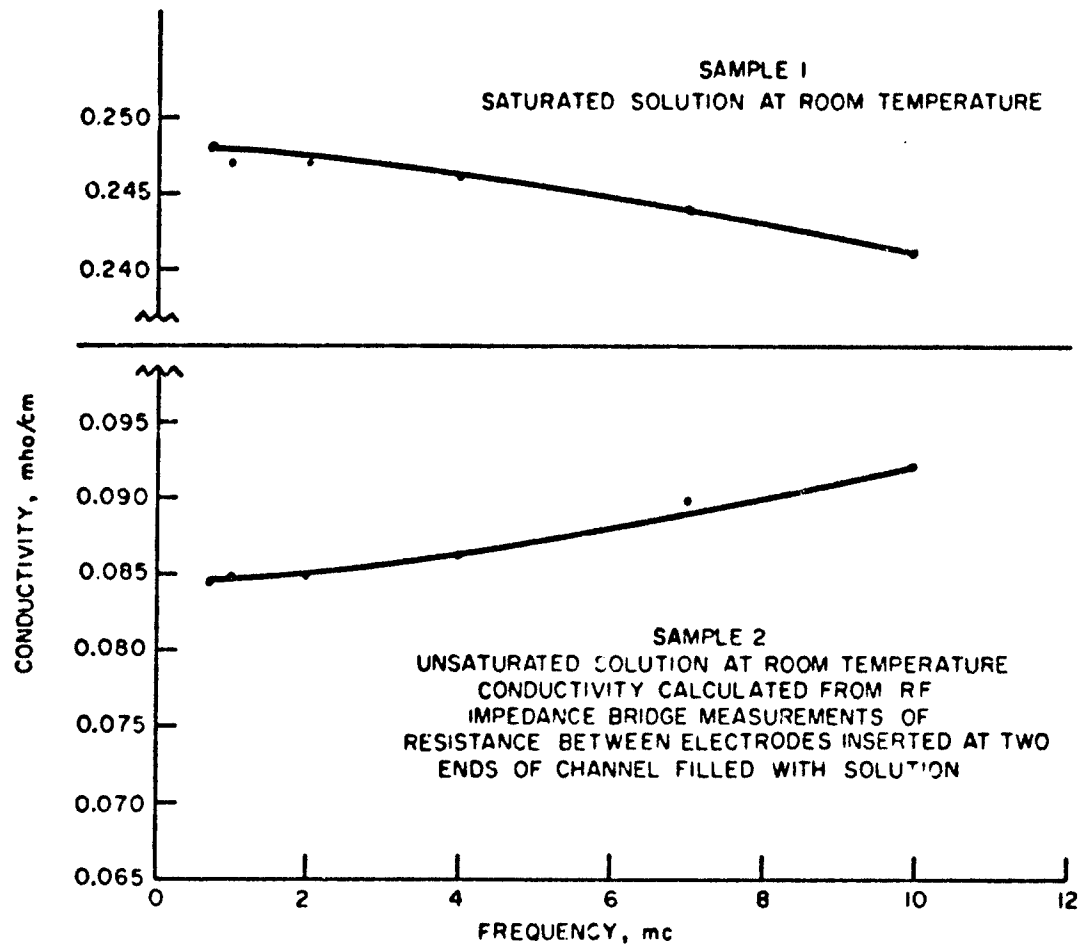


Figure 6. Salt Water Conductivity Measurement

The presence of a capacitive reactance was observed in the Q-meter readings as well, ΔX turning out to be positive. If Z_L (Figure 5) were purely resistive, ΔX would be negative. Thus the capacitive reactance introduces a problem into the conductivity measurement of the calibrating substance which is not expected to exist in the actual case of the plasma jet measurement. One could, therefore, proceed with the plasma jet measurement just on the basis of the theory. If a check is desired, a substance would have to be found which does not exhibit the capacitive effects. It is possible that fused salt or some of the iron sulphides may have the required properties for calibration of the instrument. But there was no time to pursue this matter.

Figures 8, 9 and 10 show the values of $\Delta R/\Delta X$ obtained from Q-meter measurements for tubes of salt water of various diameters and conductivities at various frequencies. The graphs show that $\Delta R/\Delta X$ decreases with increasing conductivity, as it should, but the slope is too small when compared with the simple theory. The behavior with respect to r_c , the radius of the salt water tube, does not correspond everywhere to the theory, neither does the variation with frequency. An attempt was made to take into account the capacitive reactance by assuming a circuit for Z_L consisting of a capacitor and a resistance in series, the capacitance varying slowly with frequency. When the measured reactance, ΔX , is plotted as a function of frequency, the slope is found to be negative in some cases. This is not possible; and, therefore, either this simple equivalent circuit is inadequate or the accuracy of the measurements may not be sufficient to draw significant conclusions concerning the proper equivalent circuit. The second alternative is plausible since in the calculation of the equivalent capacitance it is necessary to compute second differences of the data.

A second problem in the application of this method concerns multiple grounding. The arc jet is, of necessity, grounded at the nozzle. Parts of the coil are at relatively high RF potential with respect to the ground of the Q-meter. Thus, currents can flow through the plasma jet to ground by way of the capacitance between the coil and the jet. Such currents have appreciable effects on the Q-meter readings.



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Figure 7. Conductivity of Salt Water vs. Frequency

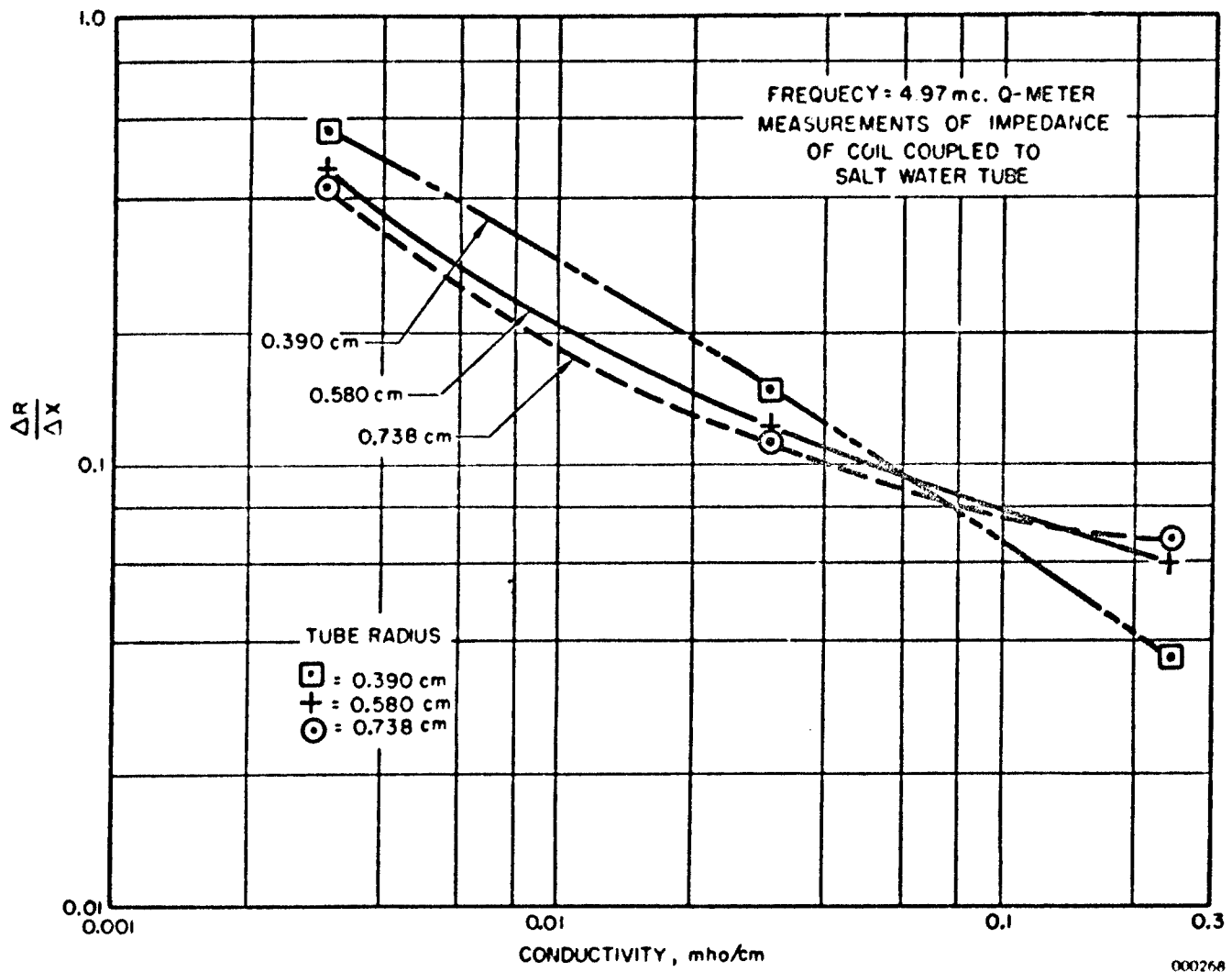


Figure 8. Ratio of Reflected Resistance to Reactance of Salt Water Tube vs. Conductivity at Constant Frequency

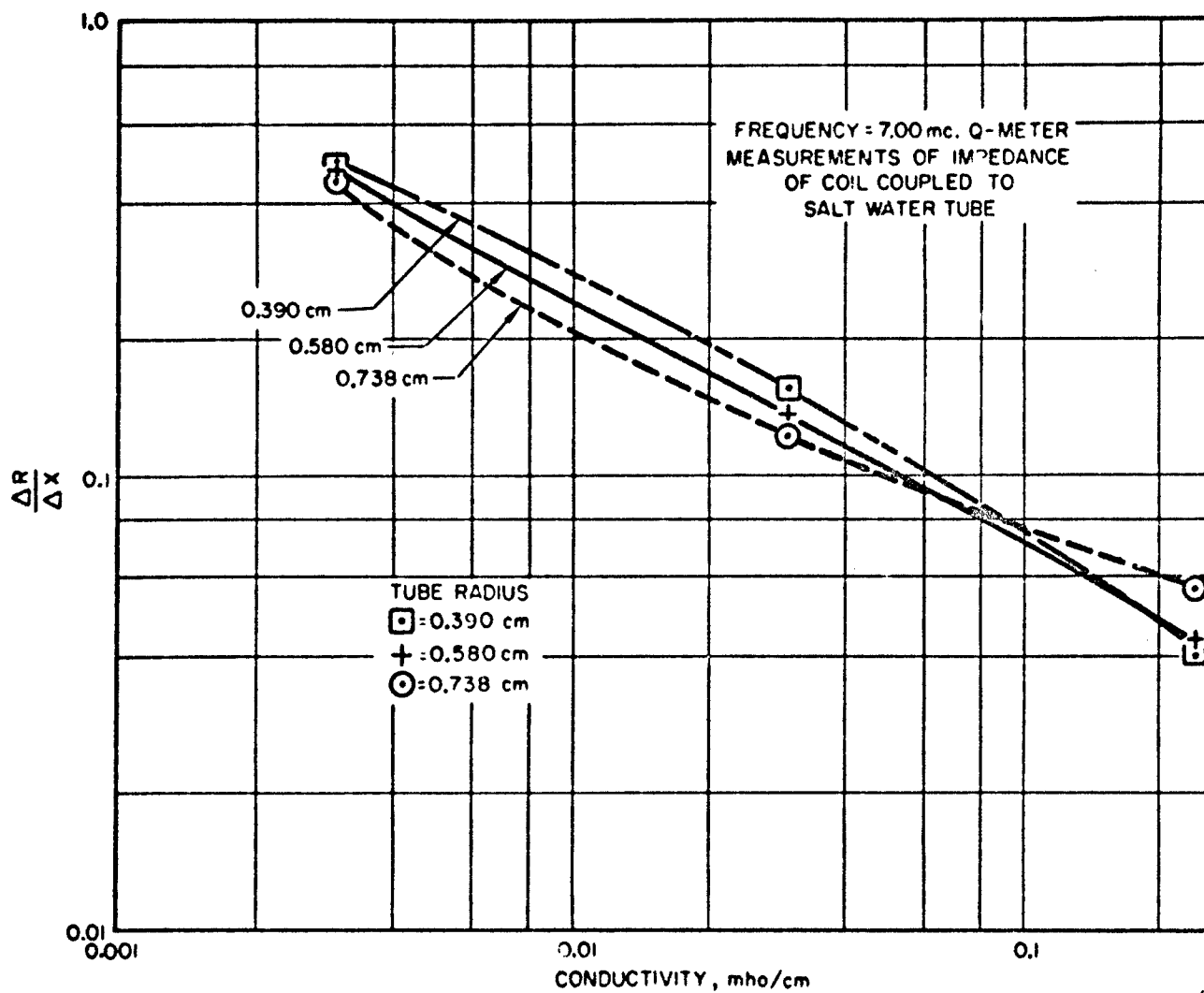


Figure 9. Ratio of Reflected Resistance to Reactance of Salt Water Tube vs. Conductivity at Constant Frequency

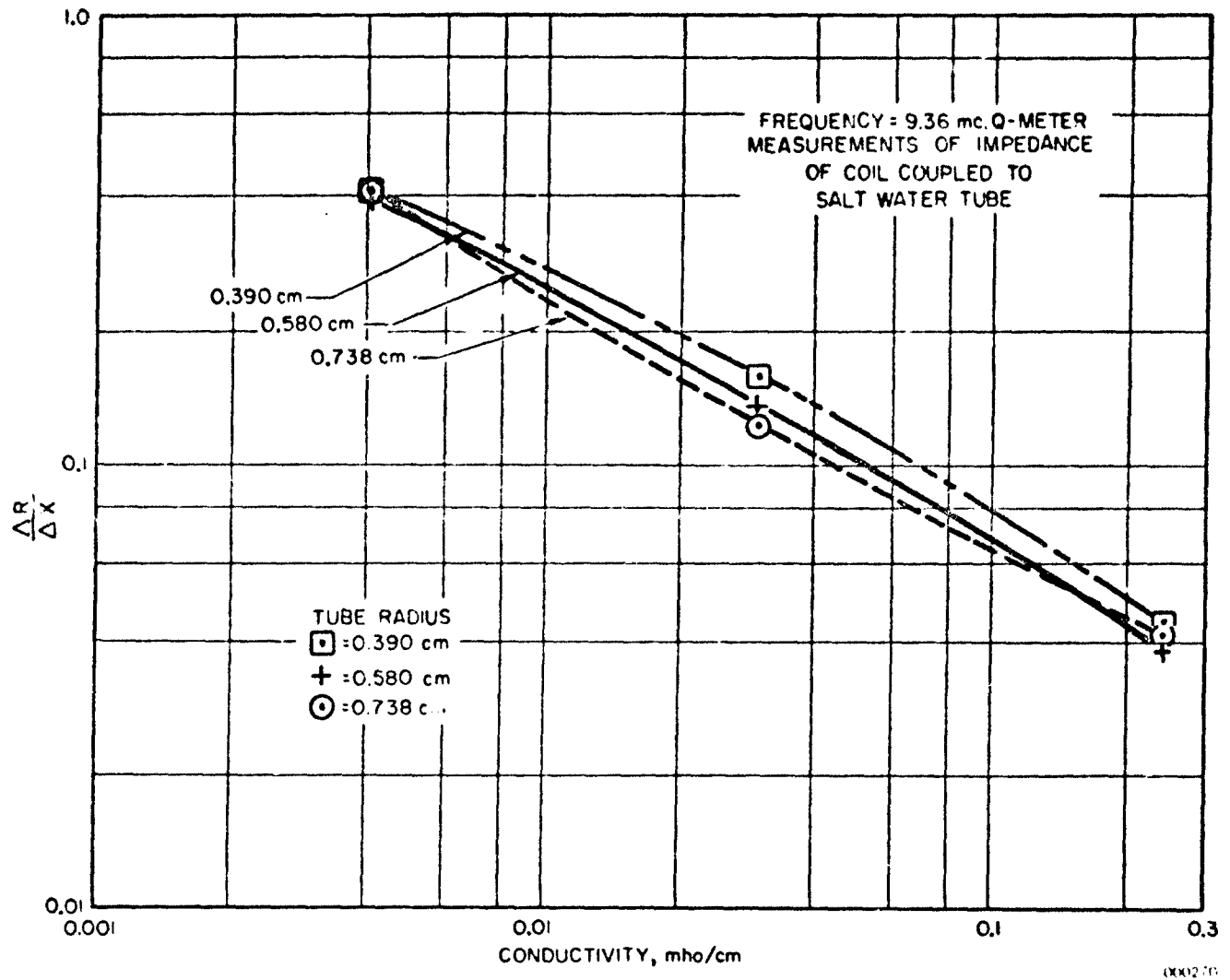


Figure 10. Ratio of Reflected Resistance to Reactance of Salt Water Tube vs. Conductivity at Constant Frequency

One solution might be the use of an RF transformer at the Q-meter (or bridge) inductance terminals. The measuring coil should then be connected to the output coil of the transformer with coaxial cable; the coils should be physically far enough apart to make direct coupling between transformer coils and measuring coil negligible. The measuring coil should have a grounded tap which can be adjusted such that the measurement is unaffected by whether or not one end of the plasma jet or the salt water tube is grounded. It is, of course, important that such an adjustment should not be sensitive to the particular way the ground connection is made at one end of the plasma jet. One should also be able to duplicate measurements without having to change the adjustment. It is likely that these conditions could be met since the plasma and coil are located inside the arc jet test chamber, and are therefore shielded from external influences.

Another solution suggested by E. Baines of the Telecommunications Laboratory is the use of a balanced measuring coil as shown in Figure

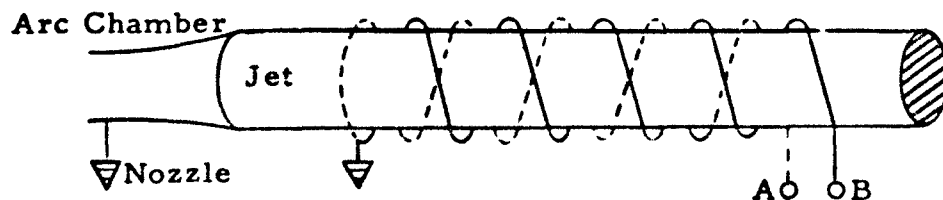


Figure 11. Balanced Measuring Technique

If a voltage is applied at AB which is balanced to ground, then currents flowing to ground through capacitive coupling cancel out at terminals AB by symmetry. One way of providing such a balanced voltage is to use a tuned push-pull amplifier. Resonance would be indicated by a minimum reading of an RF ammeter. It would also be necessary to read the RF voltage across the tank circuit. On the other hand, it may be possible to use the Q-meter, as before, with some arrangement of RF coils which transforms the unbalanced Q-meter circuit into the required balanced circuit.

It is clear that without eliminating the effects of the grounding of the plasma jet or finding a way to take them into account the measurements are without meaning. The grounding problem therefore represents an important aspect of the development of this technique for measuring conductivity.

III. ALTERNATE APPROACHES

A simple, direct method of measuring conductivity was described recently by C. P. Bean et al.³ It consists of applying an axial DC magnetic field to a rod sample of the material of interest, and recording the time decay of the voltage induced in a search coil coupled symmetrically to the rod after opening of the circuit which produces the magnetic field. This method has the advantage over both methods discussed in (II) that one measures directly the time constant of the sample, whereas in the methods of (II) a difference measurement is required. Conductivity is computed from the time constant measurement using Eq. (9).

In the application of this method to the measurement of the conductivity of an arc jet, two technical difficulties arise: (1) since the time constant for a plasma jet whose diameter is of the order of one inch is measured in millimicroseconds, the switching becomes a problem, since switching times should be small compared to the decay times of interest; (2) the time decay has to be recorded on an oscilloscope whose time response is in the millimicrosecond range. Such instruments using traveling wave amplifiers can be obtained. It remains to be seen what practical problems arise in the use of a traveling wave oscilloscope.

Thus the application of this direct method to the conductivity measurement of a plasma jet requires extensions of laboratory techniques. The grounding problem discussed in (II) is relatively easy to solve in this case, since only a voltage difference is to be measured and the potential reference is of no particular concern. However, the difficulties in calibration are the same as before (II) if salt water is used.

A list of other methods follows:

Resonant Cavity⁵

The change in the cavity resonance curve due to a jet passed through a microwave cavity can be used to measure the jet conductivity.

Microwave Beam

For wavelengths small compared to the jet diameter, the attenuation of a microwave beam crossing the jet can be used to measure the jet conductivity.

$v \times B$ Measurement⁶

The magnetic field of the plasma currents due to both the plasma motion and the transverse component of an applied low frequency magnetic field can be measured and related to the conductivity. A knowledge of the jet speed is necessary in this method.

RF Column Resistance

It should be possible to measure the resistance of a column of the jet using an RF impedance bridge. The bridge terminals are connected to two wide rings around the jet, the distance between rings defining the column length. The gap between ring and jet must be small enough so that the gap capacitance can be balanced out by the bridge.

IV. SUMMARY AND CONCLUSIONS

Transient and steady-state AC methods were considered for the measurement of electrical conductivity. The steady-state AC method was found to be superior. One problem was encountered when an attempt was made to use a saturated aqueous salt (NaCl) solution to calibrate the measurement in the conductivity range expected of an arc jet. It turned out that at the measurement frequencies such a salt solution has capacitive reactance which complicates the interpretation of the results. One possible solution to this problem is to look for other substances in the pure liquid or solid states that have the required conductivities and behave like pure resistors. Examples of such substances that may satisfy these conditions are fused salt and one or more of the iron sulphides. Another approach would be to develop measurement procedures that take into account the capacitive effects mentioned. A second problem concerns multiple grounding. This problem has to be considered only when it is desired to apply this method to the conductivity measurement of an arc jet. Again, it may be possible to eliminate this problem by appropriate circuit modifications or to solve it by making measurements in such a way as to take into account the effects due to multiple grounding.

There is no doubt that, given more time, the inductive coupling method for measuring conductivity can be made to work, and that an instrument would result which is reasonably accurate and reliable.

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